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On the Convergence of a Turbo Equalizer in terms of AMI

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***Abstract*—The average mutual information (AMI) is a useful metric to study the convergence behavior of the iteratively decodable receivers. A turbo equalizer (TEQ) has been considered as a class of such iterative receiver. We derive a new expression for the AMI at the equalizer input under no *apriori* information condition. Its value is compared with that at the output of the equalizer under perfect *apriori* information conditions. The importance of this study is to evaluate the effect of a TEQ on a signal corrupt with severe intersymbol interference and additive white Gaussian noise (AWGN). This study is useful for studying the convergence behavior of a TEQ without resorting to time consuming extensive computer simulations as usually reported in literature.**

***Index Terms*—SCS, AMI, TEQ, *apriori* information, ISI**

I. Introduction

The AMI [Bri '01] is a useful metric to study the convergence behavior of the iteratively decodable receivers. Such receivers have been studied for parallel concatenated systems and serial concatenated transmission systems. In a serial concatenated system (SCS) [1], the output of the inner decoder serves as the *apriori* for the outer decoder after suitable deinterleaving and the output of the outer decoder becomes the apriori for the inner decoder after interleaving. The SCS considered in this work is the concatenation of an outer forward error correcting (FEC) encoder and the intersymbol interference (ISI) channel. The AMI evolving at the corresponding equalizer output (the inner decoder) and that at the decoder output is studied for different conditions of the apriori information with a view to gain more insight into the operation of the turbo equalizers. The literature on turbo equalization is extensive [2-7]. However, most of the results cited in literature are based on time consuming computer simulations and only one analytical result for the equalizer output AMI is reported for the case of perfect apriori information condition at the in [8]. The complete evolution of AMI at the equalizer input and equalizer output for a particular channel at a given SNR seems to be missing. In this work, we derive the AMI at the equalizer input which is valid for a generic equalizer. The AMI at the equalizer and decoder output is simulated and compared with the analytical value by using new derived equations. The AMI evolution at the MMSE equalizer output for no *apriori* and perfect *apriori* conditions is computed in order to compare the AMI corresponding to the trellis search based equalizers and the filter based equalizers.

II. System Model

The SCS considered in this work is a closed loop system that comprises of an equalizer and a decoder. The equalizer and a decoder are joined together by means of a deinterleaver. Both the equalizer and the decoder exchange information in the form of prior probabilities of symbols. Under the assumption of AWGN at the receiver front end and perfect synchronization, the whitened matched filter [9] output samples z_k provide a set of sufficient statistic for detection purpose for a coherent symbol-spaced receiver. The z_k 's are expressed as

$$z_k = \sum_{l=0}^{L-1} h_l x_{k-l} + w_k \quad (1)$$

where h_l is the l-th tap gain of the ISI channel of length L, w_k is an i.i.d. AWGN sample which is drawn from a Gaussian distribution of zero mean and variance σ_w^2. We may also write (1) as

$$z_k = h_0 x_k + \sum_{l\neq 0, l=1}^{L-1} h_l x_{k-l} + w_k \quad (2)$$

The z_k 's along with the soft output of the decoder work on the trellis of the channel in order to compute the most likely sequence of the transmitted bits and also to compute the log likelihood ratio (LLR) on each of them. The sequence of LLRs, after deinterleaving is fed as *apriori* information to the decoder. This decoder works upon the trellis of the outer FEC encoder used at the transmitter. Similar to the equalizer, the decoder also produces another set of LLR values for all the coded bits. These LLR values, after interleaving, are fed as *apriori* to the equalizer for the next cycle of operations to begin. The LLR of a given bit at the equalizer output is defined as

$$\Lambda_{Eq}\left(x_k\right) = \log \frac{p\left(x_k = 1 | z_0 : z_{N-1}, \lambda_{Dec,0} : \lambda_{Dec,N-1}\right)}{p\left(x_k = 0 | z_0 : z_{N-1}, \lambda_{Dec,0} : \lambda_{Dec,N-1}\right)}, \forall k \quad (3)$$

where $z_0 : z_{N-1}$ is an $1\times N$ row vector of the MF outputs and similarly $\lambda_{Dec,0} : \lambda_{Dec,N-1}$ is another similar vector of the decoder soft outputs. The LLR computation as defined in (3) starts by computing the branch metric for the equalizer as

$$p(z_k \mid x_k) = \frac{1}{\sqrt{2\pi\sigma_w^2}} \exp \frac{-\left(z_k - h_{k,0}x_k - \sum_{l\neq 0} h_{k,l}x_{k-l}\right)^2}{2\sigma_w^2} \cdot \frac{\exp\left(\lambda_{Dec}(x_k)\right)^{x_k}}{1+\exp\left(\lambda_{Dec}(x_k)x_k\right)}, x_k = 0,1 \tag{4}$$

The subscript $h_{k,l}$ denotes a time-varying channel. We note from (4) that, it is the contribution from other taps $h_l, l \neq 0$ of the multipath channel that needs to be estimated and subsequently cancelled from the matched filter output z_k. We assume the main tap to be h_0 that contains the symbol of interest at the current instant. This is required so that the soft output Viterbi algorithm (SOVA) decoder observes an ISI free equivalent AWGN channel at its input asymptotically. To do so, we need knowledge about the interfering bits and the channel taps. Under the assumption of the perfect knowledge about the channel taps, then knowledge about the interfering bits is provided by the decoder.

The extrinsic information produced at the equalizer output is

$$\lambda_{Eq}(x_k) = \Lambda_{Eq}(x_k) - \lambda_{Dec}(x_k)\,\forall k \tag{5}$$

This row vector of extrinsic information of size $1\times 2K$ is deinterleaved, assuming a rate ½ outer code. The deinterleaver applies the inverse functional mapping of the interleaver and the inputs to the decoder are denoted by time instants n. The SOVA decoder accepts the vector as defined in (4) as the *apriori* on the coded bits and computes another set of LLR values as

$$\Lambda_{Dec}(c_{n,i}) = \log \frac{p\left(c_{n,i}=1 \mid \lambda_{Eq,0}:\lambda_{Eq,N-1}\right)}{p\left(c_{n,i}=0 \mid \lambda_{Eq,0}:\lambda_{Eq,N-1}\right)}, \forall n,i \tag{6}$$

where $c_{n,i}$ is the i th coded bit corresponding to the n th time instant. The decoder computes (3) by starting with the computation of the branch metrics Similar to the equalizer. The AMI is generally expressed as

$$I(\lambda;C) = \frac{1}{2}\sum_{c\in\{\pm1\}} \int_{-\infty}^{\infty} p(\lambda \mid c).\log_2\left(\frac{2p(\lambda \mid c)}{p(\lambda \mid 1)+p(\lambda \mid -1)}\right)d\lambda \tag{7}$$

where $p(\lambda \mid c) = p(\lambda \mid C = c)$ denotes a probability density function of the extrinsic information λ given that c was transmitted.

III. THE NEW AMI AT THE EQUALIZER INPUT

The ISI and the noise terms in the RHS of (2) may be combined together to define a new random variable that may be approximated as another Gaussian random variable with a mean of zero and variance given as $\sigma_{w'}^2 = \sigma_w^2 + \sum_{l\neq 0, l=1}^{L-1} h_l^2$, where σ_w^2 is the AWGN variance. Thus, the MF output samples may be approximated to be drawn from an independent Gaussian distributed random variable with a mean of h_0 and variance $\sigma_{w'}^2$. The AMI between the interleaved coded data bits $x_0 : x_{N-1}$ and the sampled equalizer input z_k may be expressed as

$$J\left(\sigma_{w'}^2\right) = 1 - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_{w'}^2}} e^{-\frac{(z-h_0)^2}{2\sigma_{w'}^2}} \log_2\left(1+e^{-z}\right)dz \tag{8}$$

where the main tap is assumed to be h_0. This expression is used to compute analytically the mutual information at the equalizer output corresponding to the no *apriori* information. The maximum mutual information (MI) produced by the equalizer corresponding to the asymptotic case of complete ISI removal is computed from the asymptotic extrinsic information at the equalizer output [10] is expressed as

$$\lambda_{Eq,asymp} = \frac{2x_x}{\sigma_w^2}\left(\sum_{l=0}^{L} h_l^2 + h_l w_{n+l}\right) \tag{9}$$

The corresponding at its output is given as

$$I_{Max,Eq}\left(\sigma^2\right) = I\left(\frac{4P_h}{\sigma_w^2}\right) \tag{10}$$

(10) follows from the fact that, under the asymptotic conditions of *apriori* information equalizer extrinsic information has a mean of $\frac{2}{\sigma_w^2}$ and a variance of $\frac{4}{\sigma_w^2}$ for normalized channel taps. This depends only on the channel SNR. It is independent of the spectral nature of channel, the channel taps and the pilot symbols. The asymptotic bit error rate is dominated by the minimum free distance of the outer code. In a similar vein, the maximum MI produced by the decoder is

$$I_{Max,Dec}\left(\sigma^2\right) = I\left(\frac{4d_{free}P_h}{\sigma_w^2}\right) \tag{11}$$

The asymptotic model used for computation of the numerical results using (8)-(10) is shown in Fig.1. In this

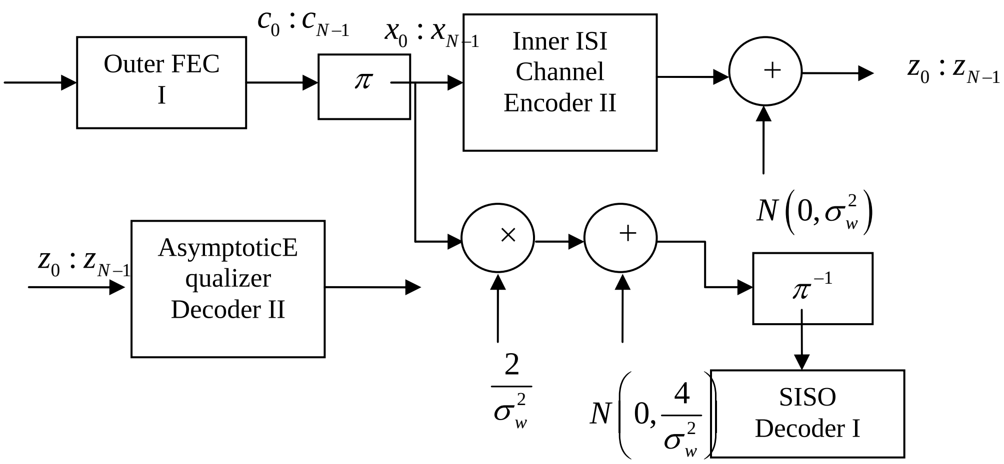

Fig.1 Asymptotic Model of the TEQ

Table No.1 Comparison of simulated and analytical AMI values

SNR (in dB)	Asymptotic AMI at equalizer output	Simulated AMI at Decoder Output	Analytical AMI at Decoder Output from (10)
3.0	0.7226	0.9693	0.9971
3.5	0.7590	0.9930	0.9987
4.0	0.7945	0.9950	0.9916
5.0	0.8590	0.9992	0.9998

figure, π represents the interleaver and π^{-1} is the deinterleaver. These two blocks are an essential component of the SCS and the interleaver renders the encoded bits statistically independent.

IV. NUMERICAL RESULTS AND DISCUSSIONS

We have considered a rate ½ recursive systematic convolutional code (RSC) that has a constraint length of 3 and feedforward generator polynomials $[7,5]_8$ while the feedback polynomial is given as $[7]_8$. The input data block size is 10000 bits. A random interleaver has been used in our work that has a size of 20000 bits. The interleaved coded data bits enter a three-tap ISI channel having the coefficients as [0.407 0.815 0.407]. This channel is known as Proakis-B channel in literature [11]. This channel has been taken as it represents spectrally a very bad channel. Previously known techniques of ISI cancellation fail short of achieving the coded AWGN bound on this channel. The ISI corrupt bits are further contaminated with AWGN at the receiver front-end. For computation of the AMI at the equalizer input, (7) has been used and the AMI at the equalizer output has been computed numerically using (8). The numerically computed value has been compared with the simulated AMI as obtained from Fig.1 in Table 1. The simulated values of post equalization and decoding AMI have been evaluated by using Fig.1. It is observed from Table 1 that, the simulated and analytic AMI values as measured at the decoder output are in close match. Both the equalizer as well as the decoder AMI is a function of the noise variance only. The SNR has been considered from 2 dB onwards due to the fact that this channel can not be equalized by the TEQ below 2 dB SNR. The AMI is in nats/symbol. We next show the evolution of the AMI from the equalizer input to its output as the SNR is varied. The observation of Table 2 shows that the AMI at the equalizer input as well as at the equalizer output increase monotonically as the SNR increases from 0 dB to 10 dB. It may be mentioned here that while the input AMI is specific to the ISI channel, the asymptotic AMI at the equalizer output is independent of the channel type. It depends only on the noise power and the constraint length of the code. The input AMI depends on the residual ISI power and the noise power as well. This table is obtained for a rate ½ code. We note that, we have used the expression for the MI between the equalizer extrinsic information and the ISI channel inputs and obtained these numerical values by directly integrating the formula as outlined in (10). It is observed that, this MI depends only on the channel SNR and it is independent of the range of the extrinsic information. This implicitly assumes the extrinsic information to be Gaussian distributed with the symmetry property. However, these values are not obtained for the initial iterations as the symmetry property is not satisfied. The mean becomes 1 and the variance is equal to the noise variance only.

Table No.2 Evolution of AMI at the equalizer input and output for different channel SNR values for two extreme *apriori* information conditions

Channel SNR	No *apriori* Equalizer Input AMI	Equalizer Output AMI
0.0	0.2131	0.4130
1.0	0.3342	0.5549
2.0	0.4246	0.6601
3.0	0.4927	0.7387
4.0	0.5446	0.7980
5.0	0.5842	0.8431
6.0	0.6148	0.8776
7.0	0.6385	0.9042
8.0	0.6570	0.9248
9.0	0.6714	0.9409
10.0	0.6827	0.9534

V. CONCLUSION

Analytical values of AMI have been derived and computed corresponding to the no *apriori* information and perfect *apriori* information in this work. We have also computed the AMI for post equalizing and post decoding conditions. The asymptotic value of the post equalizing AMI is a function of the noise variance. The AMI increases for longer constraint length codes.

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