A High Performance Wiener Filter based Turbo Equalizer

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Abstract— A turbo equalizer is a receiver that combines the tasks of an equalizer and a channel decoder in an iterative manner. The equalizer and the decoder exchange soft information called extrinsic information back and forth till a predetermined number of iterations are carried out or convergence is achieved. The equalizer is configured either as a trellis matched to the finite impulse response (FIR) nature of the ISI channel or it may be a filter that uses the minimum mean square error (MMSE) as the cost function. It has been proved in literature that, the MMSE Wiener filter based turbo equalizer is a better choice than the trellis based equalizer from the performance-computational complexity tradeoff. However, the MMSE linear filter is still computationally demanding as it requires a matrix inversion per received sample to update the equalizer coefficients per iteration. We propose a new low complex alternative to this filter. The average power of the entire block of soft decoded data is used to update the filter taps and this is done only once per block of received data per iteration. Hence, instead of inverting a matrix for every sample per iteration, the matrix inversion is done only once per block per iteration. This saves the operations by a factor proportional to the length of the received data block and the matrix inversion operation. Performance of the proposed received is evaluated by computer simulations and comparisons are made with the receiver that use the original approach.

***Keywords*— soft bit, matrix inversion, turbo**

I. INTRODUCTION

Receivers with enhanced signal processing capabilities are required in order to support multimedia services over wireless network The demand for supporting higher data rates over time-varying wireless channels is still a challenge due to their multipath nature. This multi path induces intersymbol interference (ISI) when the channel delay spread is longer than a bit or symbol duration. This ISI degrades the performance of a receiver severely by reducing its noise margin. It is not adequate to simply increase the transmitter power to take care of this problem. Hence, equalization of the received data is carried out to counteract the effect of the channel. With the advent of turbo codes[1], it has become a standard practice to employ the concept of feedback in an iterative manner to a signal processing block in a receiver. When the turbo principle is applied to an equalizer and a decoder connected by an interleaver so as to jointly mitigate (ISI) as well as random errors, the process is called turbo equalization [5-8]. The equalizer is designed to take care of ISI while the channel decoder combats the additive white Gaussian noise (AWGN). The equalizer and the decoder are configured as soft-in soft-out (SISO) signal processors. The primary objective of a SISO module is to generate the *aposteriori* probability on each bit and extract a part of information called extrinsic information. This is further fed to the other signal processor to make the decisions on the received noisy and ISI corrupted bits hopefully better with every iteration. The equalizer may be a trellis that is matched to the finite state machine (FSM) nature of the ISI channel or a conventional filter that is capable of accepting soft information as the *apriori* probability of bits. We consider the latter type of equalizer in the present work as it offers an attractive alternative to the trellis equalizers that employ the maximum *aposteriori* (MAP) [2] probability algorithms or the soft output Viterbi algorithm (SOVA)[3]. Though these equalizers are optimum from the point of minimization of bit error or sequence error, however they suffer from an intensive computational complexity requirement. The operations performed by these equalizers are of the order of $O\left(M^L\right)$ where M is the cardinality of modulation alphabet and L is the length of the ISI channel. It is obvious that, for higher order modulation alphabets or long channels, these equalizers are unsuitable. The MMSE cost function based Wiener filters that accept *apriori* information [6-8] is an attractive alternative to these trellis based equalizers from a performance-computational complexity tradeoff. This equalizer needs a matrix inversion that calls for $O\left(N^3\right)$ operations where N is the length of the filter. Hence, this also is not very attractive for practical implementation. Two approximate solutions are proposed in [6] to address this problem. The first solution computes the variance of the bits and uses a time average to update the autocorrelation function and this needs the computation of the filter taps first. The second solution assumes the availability of perfect *apriori* information which is not true when the receiver is in between the iterations. Thus, we address these problems by proposing a method that computes the power of the decoded data bits available at the decoder output at the end of an iteration and this is used to update the filter taps. This has the advantage of giving a savings in computations proportional to the entire block length of the received data frame and the matrix inversion operation.

The organization of the paper is as follows. We discuss the system model in Sec II. The proposed scheme is developed in Sec III. Results obtained by computer simulations are presented in Sec IV. Conclusions are made in Sec V.

II. System Model

The basic system model considered for our study is shown in Fig.1. An outer forward error correcting (FEC) encoder is connected to the ISI channel through an interleaver of suitable size and type. The concatenation of the analog pulse shaping filter used at the transmitter, the continuous time physical dispersive channel, the analog matched receiver filter is an analog channel. A portion of the receiver is shown in Fig.2 where π is the interleaver and $\pi^{-1}$is the deinterleaver. A digital receiver uses a sampler to sample the output of this analog channel and hence we obtain its discrete time equivalent. This receiver is modeled as per the oftcited Forney's receiver [4].A noise whitening filter (WF) used at the output of the sampler results in a whitened matched filter (WMF) that produces a set of sufficient statistics for detection. Thus, the overall combination of the noise WF and the sampler results in the discrete time transversal filter (DTTF) $h_l; l = 0 : L-1$ (called the channel taps) that represents the ISI channel. Without any loss of generality, we assume one sample per bit by considering the transmitted signal to be strictly band limited to $1/2T$ Hz where T is the symbol period. Thus, no excess band width is allowed. The ISI channel is modeled as a finite state machine (FSM) [9] to which MAP and VA algorithms are applied. The baseband time domain low-pass WMF samples as observed at the output of a one-step Gauss-Markov process are expressed

$$z_k = \sum_{l=0}^{L-1} h_l x_{k-l} + w_k \tag{1}$$

assuming bit rate sampling at the receiver and w_k is an i.i.d. AWGN sample drawn from a Gaussian distribution $N\left(0, \sigma_w^2\right)$. The receiver shown in Fig.2 uses a SISO equalizer as the inner decoder (decoding and detection carry identical meaning here). This has two inputs; i.e. the vector of channel observations $z_0 : z_{K-1}$ and the *apriori* information on all the coded symbols. This second information comes from the decoder. Initially, the *apriori* information is identical for all the bits and the equalizer produces a vector of K nuber of LLRs by making use of the FSM nature of the ISI channel. The soft output is calculated by assuming the filtered outputs to be Gaussian distributed with a mean and variance that depends on the filter coefficients. The extrinsic information about the coded bits is derived by subtracting the apriori information from the LLR. This is deinterleaved and fed as *apriori* for the decoder. The decoder makes use of the FSM matched to the outer FEC and computes a set of updated LLR values for the same set of coded bits. Another set of extrinsic information is computed at the decoder output. The decoder's extrinsic information is interleaved and fed as *apriori* to the equalizer. This process of feeding extrinsic information between the equalizer and the decoder is repeated a predetermined number of times or convergence is achieved. At the end of the iterations, hard decision on the decoder output is taken. In a turbo equalizer, we estimate all the K number of coded interleaved bits $x_0 : x_{K-1}$. The equalizer coefficient determination reduces to the classical problem of minimization of error variance between x_k and $\hat{x}_k$ which is stated as

$$J = E\left[\left|x_k - \hat{x}_k\right|^2\right] \tag{2}$$

III. COEFFICIENTS OF THE EQUALIZER IN TERMS OF *APRIORI* INFORMATION

The estimate $\hat{x}_k$ of the data bit x_k at the output of the equalizer is written in terms of its coefficients $\mathbf{c}_n$ as

$$\hat{x}_k = \mathbf{c}_k^H \mathbf{z}_k \tag{3}$$

$$\mathbf{c}_k = \left[c_{k,N_2}, \ldots c_{k,0}, \ldots c_{k,N_1}\right], \mathbf{z}_k = \left[z_{k-N_2} z_{k-N_2+1} \ldots z_{k+N_1}\right]^T$$

and $(.)^H$ represents Hermitian transpose. We note that, the equalizer takes care of N_2 number of postcursors and N_1 precursors. By substituting (3) in (1) and performing the minimization of

$\frac{\partial J}{\partial \mathbf{c}_n} = 0 \; for \; n = 1, \ldots \left(N = N_1 + N_2 + 1\right)$, we solve a set of N linear equations that give the filter coefficients as

$$\mathbf{c}_k = \left(R_{\mathbf{z}_k \mathbf{z}_k}\right)^{-1} R_{\mathbf{z}_k x_k} \tag{4}$$

This is the classical solution to the MMSE filter tap estimation that does not take into account the *apriori* probability of the transmitted symbols [10, p.362,E7.13]. The proposed equalizer does take into consideration the apriori as shown in (8) in the computation of both the matrices of (4).The equalizer input signal's covariance matrix is $R_{\mathbf{z}_k \mathbf{z}_k}$ while the cross correlation between the desired response and the input signal is $R_{\mathbf{z}_k x_k}$. The matrix $R_{\mathbf{z}_k \mathbf{z}_k}$ is expressed as

$$R_{\mathbf{z}_k \mathbf{z}_k} = \left(\sigma_w^2 I_N + H \mathbf{V}_k H^H\right) \tag{5}$$

It is in $\mathbf{V}_k$ that the *apriori* information about the coded bits is used and I_K is an identity matrix. H is a $N \times \left(N + L - 1\right)$ Toeplitz matrix that contains time shifted replicas of the channel taps which is assumed to be known for the present work and $V_k, k = 0, 1, \ldots N-1$ is an $N + L - 1 \times \left(N + L - 1\right)$ diagonal matrix containing the power of each bit $v_k = \bar{x}_k^2$ and

$$\bar{x}_k = \tanh\left(\frac{L_{dec}\left(x_k\right)}{2}\right) \tag{6a}$$

Thus,

$$\mathbf{V}_k = diag\left(v_{k-L-N_2+1} \, v_{k-L-N_2+2} \ldots 1 \; v_{k+1} \ldots v_{k+N_1}\right) \tag{6b}$$

The quantity $L_{dec}\left(x_n\right)$ is available from the decoder after one iteration and this serves as *apriori* information about all the coded bits to the equalizer. Precisely, this is the information used to update the $\mathbf{V}_k$ matrix for each value of k and hence the equalizer coefficients change. This use of apriori makes it different from the conventional MMSE Wiener filter. The cross correlation $R_{\mathbf{z}_k x_k}$ is expressed as

$$R_{\mathbf{z}_k x_k} = E\left[\left(\mathbf{z}_k - \bar{\mathbf{z}}_k\right)\left(x_k - \bar{x}_k\right)^H\right] = Hs \tag{7a}$$

and $s=\left[0_{1\times N_2+L-1}\,1\,0_{1\times N_1}\right]^T$ (7b)

where $(.)^T$ denotes transpose. Use of (6b) is made in the original approach of the *apriori* aided turbo equalizer.

We propose to approximate

$$\mathbf{V}_k \text{ as } \mathbf{V}_k = \frac{1}{K}\sum_{k=1}^{K} v_k \qquad (8)$$

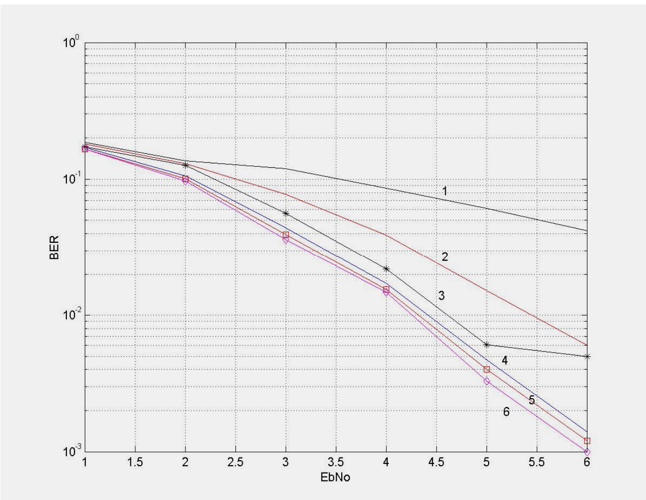
A similar scheme has been proposed in [6] in order to realize a low complex SISO filter for the equalizer. However, our scheme is different in the sense that, we use the power of the soft bit estimates to approximate the

$$\mathbf{V}_k = \mathrm{var}(\hat{x}) I_N \text{ and } \mathrm{var}(\hat{x}) = \frac{1}{K}\sum_{k=0}^{K}(\hat{x}_k)^2$$

Hence, the proposed receiver is different from [6] in three aspects: 1) A random interleaver is used instead of a S-random interleaver, 2) A traceback SOVA decoder is used to achieve one step of complexity reduction while a BCJR MAP has been used as the SISO decoder in the reference and 3) approximating $\mathbf{V}_k$ as in (8) towards a low complex filter based equalizer.

IV. RESULTS AND DISCUSSIONS

We evaluate the performance of the proposed FIR filter based turbo equalizer through computer simulations. The outer code is a recursive systematic convolutional (RSC) rate ½ code of constraint length 3 with generator polynomials [7,5], the interleaver is random and the ISI channel considered is the strongly frequency selective Proakis-B with [0.407 0.815 0.407] that causes 100% ISI. The filter is a $N=9$ tap filter so that, the proposed receiver makes a matrix inversion for this 9-tap filter for each frame per iteration. The decoder is the traceback SOVA decoder with a trace back depth of 30 bits. The average bit error rate for 6 iterations is shown in Fig.3.



1:6 6 Iterations of equalization and decoding

Fig.3 BER Performance of the low complexity TEQ with respect to 6 iterations

The proposed receiver makes a matrix inversion for this 9-tap filter for each frame per iteration. The decoder is the traceback SOVA decoder with a trace back depth of 30 bits. This receiver seems to approach a BER of 10^{-3} at an 6.0 dB SNR.

The mean square error obtained at the output of the decoder is shown in Fig.4.

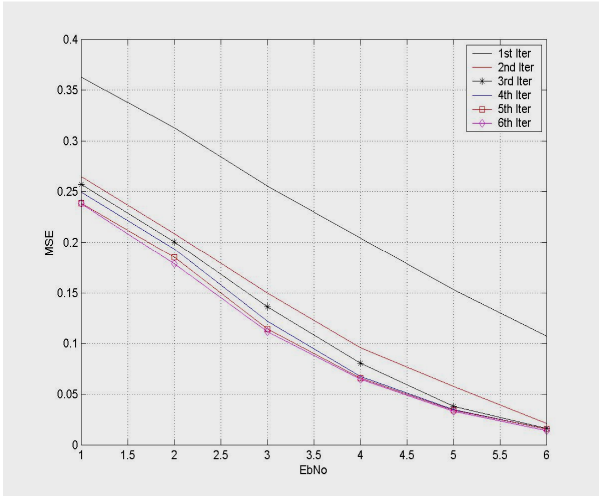


Fig.4 MSE Performance of the proposed scheme

We note that, the proposed TEQ reaches a MSE floor of about 0.01 at the end of the 6^{th} iteration at 6.0 dB.

The evolution of the average mutual information (AMI) at the decoder output of the proposed TEQ is illustrated in Fig.5. The asymptotic MI values are 0.9971, 0.9987 and 0.9998 for SNR values of 3.0 dB-5.0 dB respectively. Because of the approximation, we do not use all the available information about the coded bits and hence the average MI is less than the asymptotic values for all the SNRs shown in Fig.5.

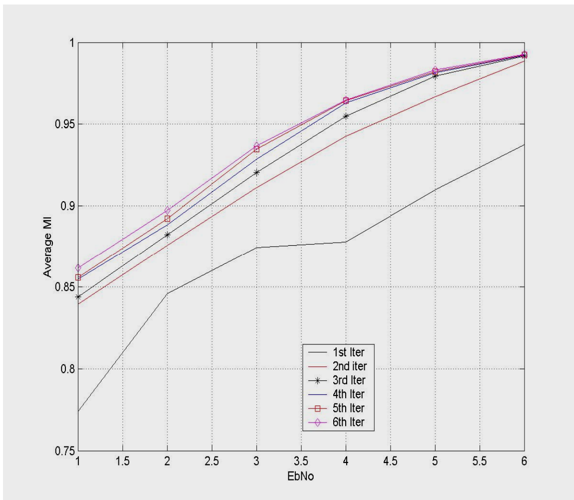


Fig.5 Evolution of AMI at the decoder output for the 6 iterations at 6 different values of SNR

We next show the average BER performance as obtainable by the use of the actual filter that uses a matrix inversion for every symbol present in the received data frame. We have considered a input data frame size of $K = 10000$ bits and 10 simulations were performed in obtaining the results shown here.

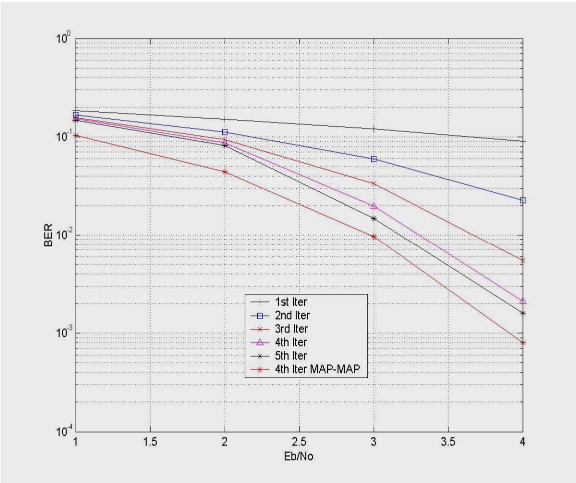


Fig.6 The first 5 curves give the performance of the actual MMSE Wiener filter that requires matrix inversion for each symbol. The leftmost curve represents the performance of the MAP equalizer-MAP Decoder Scheme for the same set up at the 4th iteration

The sample-by-sample MMSE filter seems to require about 4.2 dB SNR to achieve a BER of 10^{-3}. The tradeoff between the proposed receiver and the receiver of [2] is a savings in computational overhead by a factor $10000 \times o(729)$ while requiring an additional SNR of 1.8 dB at a BER of 10^{-3}.

The results (Fig.s 3-4) indicate that the proposed receiver is capable of approaching the performance obtainable by the more complex structures like the actual filter and the MAP-MAP receiver at the cost of a higher SNR while reducing the complexity that is proportional to 10000 times the matrix inversion per sample in the present case.

V. CONCLUSIONS

The results (Fig.s 3-5) indicate that the proposed receiver is capable of approaching the performance obtainable by the more complex structures like the actual filter and the MAP-MAP receiver at the cost of a higher SNR while reducing the complexity that is proportional to the block size $10000 \times o(729)$, and 10000 additions in the present case.

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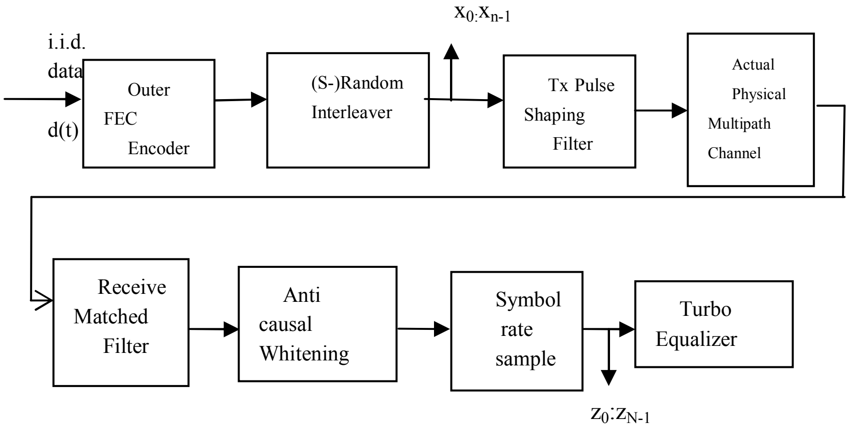

Fig.1 Portion of a Typical Coded Transmission Scheme

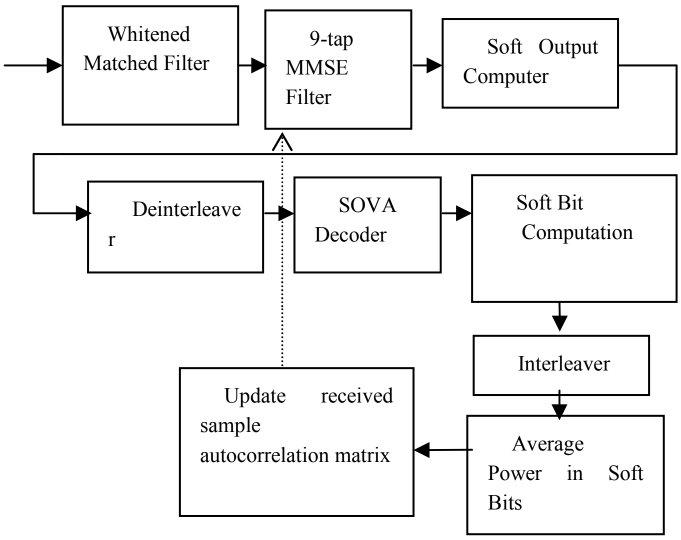

Fig.2 The proposed receiver